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SOF'YA VASIL'YEVNA KOVALEVSKAYA (ON THE 150TH ANNIVERSARY OF HER BIRTH)†

Crowned with well deserved glory, the name of Sof'ya Kovalevskaya will always remain in science and in the history of social progress in Russia.

P. Ya. Kochina (1981)

Many articles, books and memoirs are associated with our remarkable compatriot Sof'ya Vasil'yevna Kovalevskaya. Her name is widely known throughout the world. She lived a short but colourful life having left us the legacy of her remarkable papers in the fields of mathematics and mechanics. She proved, by the example of her life, the strength of women as scholars and independent members of society.

"I feel that I am destined to serve truth and science, and to build a new road for women because this means to serve justice" was the manner in which the young Sonya imagined her future life when she was 23 years old.

Kovalevskaya was born on the 3rd (15th) of January 1850 in Moscow into the family of Vasilii Vasil'yevich and Elizaveta Fedorovna Krukovskii who, in 1858, had been elevated to the rank of the hereditary nobility in the Russian empire with the conferment on them of the surname Korvin-Krukovskii. Her father, who was an artillery officer and a participant in the Russo-Turkish war of 1828–1829, retiring with the rank of lieutenant-general, took up residence with the family on his estate in the village of Palibino in the province of Vitebsk, where Sonya spent her childhood years. Her mother was the granddaughter of a full member of the St Petersburg Academy of Sciences, the astronomer and mathematician F. I. Schubert and the daughter of an honorary member of the Academy F. F. Schubert who worked in the field of geodesy. In the words of the Swedish writer, Ellen Key, Kovalevskaya later said "I inherited a passion for science from an ancestor, the Hungarian king Mathias Corvin, a love of mathematics, music and poetry from my mother's grandfather on the paternal side and a love of vagrancy and an inability to obey accepted customs from a gypsy great-grandmother: the rest was inherited from Russia".

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Her uncle Piotr Vasil'yevich Korvin-Krukovskii played a major role in stimulating her interest in mathematics while still a child. He frequently talked with her, instilling in her, according to her words "a reverence towards mathematics as the highest and mysterious science which opens a new and wonderful world to those who are dedicated to it, which is inaccessible to mere mortals". The Weltanschauung of Kovalevskaya was laid down, to a significant degree, by the influence of her elder sister, Anna, who had entered into the circle of the nihilists of the 1860s and, subsquently, participated in the events of the Paris Commune. The 17-year-old Sonya passionately strived to become independent and to broaden her education. However, her dream was impossible to realize in Russia where the doors of the universities were closed to women. She therefore left in 1869 to study in Heidelberg, having previously entered (in 1868) into a fictitious marriage with V. O. Kovalevskii, who is known for his work in the field of palaeontology, translation of the works of Charles Darwin and, also, for his publishing activities. In the Heidelberg University, she attended the lectures of Leo Königsberger and Paul Du Bois-Reymond on mathematics, the physics lectures of Gustav Kirchhoff and the physiology lectures of Hermann Helmholtz, her remarkable talents immediately attracting the attention of her teachers.

However, none other than the elderly Karl Weierstrass who at this time was professor at the Berlin University became her science teacher. In 1870, the 20-year-old Kovalevskaya began to study mathematics with him in a particular way, as the University Council had not permitted her to attend lectures. The moving friendship between the master and his favourite pupil, which Kovalevskaya became shortly afterwards, lasted until the end of her days. A total of 88 letters from Weierstrass to Kovalevskaya have been preserved and a significant number of these letters were published by P. Ya. Kochina in 1973. Weierstrass greatly valued the mathematical ability of his pupil, noting that he "had very few pupils who could compare with her as regards diligence, talents, painstakingness and enthusiasm for science".

During the years in which Kovalevskaya was directly instructed by Weierstrass (1870–1874), she carried out scientific research in three areas. Her first paper, "On the theory of partial differential equations", contained a proof of a theorem of the existence of a holomorphic solution of partial differential equations of normal form. This theorem had been formulated and proved for the first time by Cauchy in 1842 in the case of a linear system. However, Kovalevskaya's proof, which makes use of Weierstrass majorizing functions, is simpler than Cauchy's proof. It is distinguished by its completeness and is still included in courses on mathematical analysis. Her second paper, "Addenda and remarks on Laplace's investigation on the shape of the Saturn ring", is associated with the investigation of new cases of the solution of Laplace's problem on the shape of a massive, rotating, homogeneous ring of elliptic cross-section with a thin layer of a homogeneous fluid on its surface. This fluid is attracted by the mass of the ring and by the central body, the centre of mass of which is coincident with the centre of the ring and must be in equilibrium on the surface of the ring. Unlike Laplace, who had investigated this problem under the assumption that the ring diameter was many times the size of its elliptic cross-section, Kovalevskaya constructed a particular solution of the problem which did not impose any constraints on the relation between the dimensions of the cross-section and the ring diameter. Her third paper, "On the reduction of one class of third rank Abelian integrals to elliptic integrals", was a development of the work of Weierstrass and was concerned with the theory of Abelian functions; in particular, she investigated a hyperelliptic integral which contains a polynomial of eighth degree as a radicand.

In 1874, the Göttingen University awarded Kovalevskaya the academic degree of Doctor of Philosophy "summa cum laude".

In the same year, the Kovalevskii's, who had often lived apart abroad, returned to their homeland. They easily associated with the circle of academics and writers of that time which, in particular, included D. I. Mendeleyev, I. M. Sechenov, A. M. Butlerov, P. L. Chebyshev, A. V. Gadolin, I. S. Turgenev and F. M. Dostoyevskii. "Sonya immediately became the focus of one of these élite, intelligent circles which were highly devoted to intellectual interests, were a feature of the Russian capital and rarely met at any other place in Europe", wrote Anna-Charlotte, sister of Professor Gösta Mittag-Leffler in her memoirs of Kovalevskaya. During these years of life in her homeland, Kovalevskaya contributed to the newspaper *Novoye Vremya* (*New Time*). In 1876 and 1877, she published four scientific reviews dealing with the use of solar energy and problems of optics, aeronautics, the invention of the telephone and talking telegraph, fermentation processes and the essence of enzymes. In addition, she wrote ten reviews of theatrical performances of the 1876/1877 season.

Gradually, her relations with V. O. Kovalevskii turned from a fictitious arrangement into a real marriage and, in 1878, a daughter, also Sonya, was born to them. As the daughter writes in her memoirs: "My first recollections of mother are associated with moving to new residences by train, with a trunk from which they took out a spirit-lamp and saucepans in which they boiled milk and cooked semolina. Mother herself is affectionate and anxious, frequently kisses me and, then puts me to bed and takes my temperature". In these memoirs, Sonya, the junior, also remarks that their relationships with her

mother over the whole of her life were complicated and not distinguished by their intimacy. Kovalevskaya strived to bring up her daughter as a strong and energetic person and "despaired when she didn't succeed". Since the mother was often moving from one city to another, the daughter spent part of her life in the families of relatives and acquaintances.

During the first years of their life back in their homeland, the Kovalevskii's were occupied with troubles concerning the material welfare of the family. So, on moving from St Petersburg to Moscow (1880), together with P. N. Yablochkov and Yu. V. Lermontova, she was occupied with the organization of street lighting. During this time, she was unable to get involved in mathematics to a sufficient extent. However, the Sixth Congress of Russian Natural Scientists and Physicians was held in St Petersburg in the winter of 1879/1880 at which, at the suggestion of P. L. Chebyshev, she presented a paper on investigatons she had carried out on Abelian integrals. Apparently, this event had an important influence on her decision once again to become deeply engaged in mathematics. However, there was no possibility for her to carry out scientific research at the universities of Moscow or St Petersburg in spite of her brilliant mathematical gifts and she was not even admitted to the master's degree examination in the Moscow University.

In the spring of 1881, she left for Berlin in order to continue her scientific activities. However, she was unable to start work immediately owing to worries associated with her relationship with her husband, which had become very complicated, and with a disquieting financial situation for the family. On the 11th of April 1882, Weierstrass, in replying to Kovalevskaya's letter, wrote "From each line of your letter and, even more so, by reading between the lines, it is quite clear that, for reasons you have not wished to mention and could not say in greater detail, you have become agitated and troubled, and this has threatened for a long time to hamper your great wish to devote yourself peacefully to your work". In the summer of 1881, Kovalevskaya received an invitation from the professor of the Higher School (University) in Stockholm, Gösta Mittag-Leffler, a well-known mathematician who had also been taught by Weierstrass, to accept the position of reader or professor in the Department of Mathematics of the Stockholm University. However, she decided not to accept this position immediately. The future destiny of her family worried her. In a letter to her husband she asks "Should I accept the invitation or not? It is sad constantly to live alone but, meanwhile, it is a great honour". However, the material position subsequently deteriorated to such an extent that she was compelled to think about the welfare of herself and her daughter. In April 1883, her husband tragically passed away, unable any longer to withstand the torture of failures in commercial matters and the ruin which had overtaken him.

In the autumn of the same year, Kovalevskaya left for Stockholm and, from February 1884 during the course of the spring semester at the Stockholm University, gave her first special course in mathematics on partial differential equations. In a letter to her brother-in-law in the spring of 1884, she wrote: "My lectures, of course, cause me a lot of trouble. I try with all my power to present them well and clearly. Sometimes I succeed in doing so and I am then very happy, but sometimes work doesn't go so smoothly and I note that I have not succeeded in interesting my students and presenting everything to them in a clear light, and this causes me great distress". From the autumn of 1884, she was appointed a professor of the Stockholm University and worked there for a period of eight years and, in 1889, was confirmed in this position for life. During these years, she gave 12 courses which included the theory of algebraic, Abelian, elliptic and Weierstrass theta-functions, the theory of motion of a rigid body, the application of analysis to the theory of numbers, a course of lectures on curves defined by differential equations, according to Poincaré, etc. Her lectures were a great success, and she herself occupied an honoured position in society. On the centenary of her birth, the Swedish newspaper *Svenska Dagbladet* testified that "the first female professor in Sweden made a dazzling impression on the inhabitants of Stockholm in the Eighties".

During the Stockholm period of her life, Kovalevskaya actively collaborated on the journal *Acta Mathematica*, was a member of its editorial board and resumed her active scientific research. During this period, she carried out an outstanding investigation of the rotation of a rigid body about a fixed point. To the well-known particular solutions of the problem of the rotation of an arbitrary rigid body about a fixed point, obtained by Euler and Lagrange, she added a third case of integrability when the principal moments of inertia of the body satisfy the conditions $A = B = 2C$ and the centre of gravity of the body lies in the equatorial plane of the ellipsoid of inertia. In this case, she discovered a fourth integral of the equations of motion (in addition to the three known integrals). In order to integrate the equations of motion, she used the theory of functions of a complex variable. The case she considered deservedly carries her name. She also formulated a theorem on the existence of just three cases of the integrability of the equations of motion in the class of meromorphic functions which are, namely, the Euler and Lagrange cases and the one she discovered. A rigorous proof of this theorem was subsequently given by G. G. Appel'rot and A. M. Lyapunov.

Her investigations made a major contribution to the theory of the motion of a rigid body. Using the

results, she published three papers. The first paper, "On the rotation of a rigid body about a fixed point", was awarded the prestigious Borden prize of the Paris Academy of Sciences. On presenting this prize on the 24th of December 1888, the president of the meeting, Jules Janssen noted that the Academy "discovered in this paper not only evidence of an extension and profound knowledge but also the sign of a mind of great inventiveness". In 1889, she was awarded the prize of the Stockholm Academy of Sciences for her second memoir on the rotation of a rigid body, "On a property of the system of differential equations, governing the rotation of a rigid body about a fixed point".

Throghout her whole life and, particularly, during the Stockholm period, when Kovalevskaya had been at the pinnacle of her fame, her great desire had been to return to her native land and to work in one of the Russian universities. However, a petion of her cousin, governor of the Saratov province, A. I. Kosich, "to return Kovalevskaya to Russia and Russian science" was followed by a negative response of the Academy of Sciences, which emphasized that chairs at Russian universities were not open to women irrespective of their capabilites and knowledge. However, a session of the St Petersburg Academy of Sciences on the 4th of November 1889 resolved to change the tradition and henceforth to admit members of the female sex to be elected as corresponding members of the Academy of Sciences. On the basis of recommendations presented by Academicians P. L. Chebyshev, V. G. Imshenetskii and V. Ya Bunyakovskii, Kovalevskaya was elected at the meeting of the Physico-Mathematical Division of the Academy of Sciences on the 7th of November 1889 and was subsequently confirmed by the General Assembly of the Academy as a corresponding member of the St Petersburg Academy of Sciences.

Kovalevskaya died before her time, from pneumonia, on the 10th of February 1891, at the age of 41. On her grave in Stockholm there is a tombstone of black granite which was erected in 1896 and paid for with money collected by the Committee of the St Petersburg Higher Women's Courses and other organizations. The German mathematician Leopold Kronecker wrote in her obituary that Kovalevskaya "in conjunction with an exceptional talent, had left the memory of her individuality, which was both considerable and, moreover, full of charm, in the hearts of all who remained and had the pleasure of knowing her".

Her talent was not only revealed in her scientific research but, also, in her literary activities. She wrote several stories, sketches and dramas. Among these, the best known are the stories "Childhood Memories" and "The Lady Nihilist". The journal *Severnyi Vestnik*, in describing the overall characteristics of her literary creativity, wrote that "in science, Kovalevskaya was a fully recognized major figure, while in Russian literature she was a figure of "bright promise". Her literary creativity is reflected in the collection of her works of literature published by the Academy of Sciences in 1974.

Throughout her life Kovalevskaya supported girls who wished to broaden their education and to continue their studies abroad. In particular, Yu V. Lermontova, who obtained a doctorate at the Göttingen University, A. M. Yevreinova, the first female lawyer in Russia, and Ye. F. Litvinova, who received a doctorate in mathematics from the Bern University, experienced her influence on them. She was the radiant point to which girls who wished to study were drawn.

She gained great respect and authority among scholars throughout the world. She was in close contact with many Russian mathematicians, including P. L. Chebyshev. A. V. Vasil'yev and D. F. Selivanov, helped young students from Russia and promoted the publication of their papers in *Acta Mathematica*. She was known by, met and corresponded with the leading mathematicians of Western Europe, K. Weierstrass, L. Kronecker, H. Schwartz and K. Runge in Germany, J. Bertrand, Ch. Hermite, G. Darboux and H. Poincaré in France, and H. Gyldén, G. Eneström, A. Lindstedt, I. Bendixson and E. Phragmén in Sweden. A close friendship also developed with the Swedish mathematician G. Mittag-Leffler who had always taken great care of her, particularly during the first years of her life in Sweden. I. Fredholm was a student of hers at the Stockholm University.

Another outstanding Russian woman, Pelageya Yakovlevna Kochina, who studied the life and activities of her great predecessor for more than half a century, was one of her greatest admirers. Thanks to Kochina's numerous papers, written with love and based on careful historical-scientific investigations, we know much more about Sof'ya Kovalevskaya than was known to previous generations.

Her fame has not dimmed with the passing years. Her service is universally recognized. It is symbolic that Kovalevskaya was included among the ten greatest academics – applied mathematicians of all time and as the sole representative from the 19th century on the emblem of the Twentieth International Congress of Theoretical and Applied Mechanics which is to take place this year in Chicago.

I. G. Goryacheva and G. K. Mikhailov

Translated by E.L.S.